THEORY OF NONSTEADY PROPELLANT COMBUSTION. COMBUSTION IN THE PRESENCE OF HARMONICALLY VARYING PRESSURE

Yu. A. Gostintsev, L. A. Sukhanov, and P. F. Pokhil

UDC 536.46

It is shown that in order to construct a theory of nonsteady propellant combustion it is necessary to know the steady-state dependences of the burning rate $u_0°$, surface temperature $T_S°$, and flame temperature $T_F°$ on the external parameters and the initial temperature of the propellant. The combustion processes in an unbounded space, when one of the external parameters varies according to a harmonic law, are examined within the framework of such a theory.

1. The theory of nonsteady propellant burning is based on the assumption that all the processes in the gas-phase and in the solid-phase reaction zone are quasi-stationary [1-3]. In this theory the only inertial region is the solid-phase induction zone, whose reconstruction also determines the nonstationarity of propellant combustion. The quasi-stationarity assumption is justified, if the characteristic times τ_1 of variation of the parameters controlling combustion (pressure, gas velocity at the burning surface, flow of radiant energy from the gas to the propellant, etc.) are much greater than the relaxation times of the processes in the gas phase and the solid-phase reaction zone. Order-of-magnitude estimates for ordinary combustion conditions give $\tau_1 > 10^{-4}$ sec.

Within the framework of these assumptions the instantaneous values of the burning rate u, the surface temperature T_S and the flame temperature T_F depend only on the n values of the external parameters z_j and the temperature gradient $\varphi_S = (\partial T/\partial x)_S$ at the surface in the direction of the solid phase

$$u = u(\varphi_s z_1, \ldots, z_n),\ T_s = T_s(\varphi_s, z_1, \ldots, z_n),\ T_F = T_F(\varphi_s, z_1, \ldots, z_n) \tag{1.1}$$

In order to demonstrate the validity of (1.1), we write the relations controlling the nonsteady combustion process in functional form. In the gas and in the thin solid-phase reaction zone these relations are the same as for steady-state burning (the quasi-stationarity condition is assumed to be satisfied). Then in symbolic form [2] the mass burning rate in the gas

$$m_g = m_g(T_F, \varphi_g, q_g, z_1, \ldots, z_n) \tag{1.2}$$

The rate of decomposition of the condensed phase (in particular, the law of pyrolysis)

$$m_s = m_s(T_s, \varphi_s, q_s, \varphi_g, z_1, \ldots, z_n) \tag{1.3}$$

The heat release in the solid-phase reaction zone q_S and in the gas q_g

$$q_s = q_s(T_s, \varphi_s, z_1, \ldots, z_n) \tag{1.4}$$

$$q_g = q_g(T_F, \varphi_g, T_s, z_1, \ldots, z_n) \tag{1.5}$$

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 60-69, September-October, 1971. Original article submitted December 18, 1970.

From the quasi-stationarity condition for the gas there follows

$$m_g = m_s = m \tag{1.6}$$

In addition, we have the energy-conservation equations for the surface

$$\lambda_g \varphi_g - \lambda_s \varphi_s = -m_s q_s \tag{1.7}$$

and for the inertialess zone as a whole

$$m(c_s T_s + q_s + q_g) - \lambda_s \varphi_s = m c_p T_F \tag{1.8}$$

It should be emphasized that here the net reaction energy $q_s + q_g$ is variable.

It is clear that in the general case the eight unknown functions m_g, m_s, T_F, T_s, φ_g, φ_s, q_g, q_s [all the $z_n(t)$ are assumed to be known] are related with each other by seven equations; accordingly, relations (1.1) hold, their form being the same for both steady-state and nonsteady combustion. Under nonsteady conditions the temperature gradient φ_s must be determined from the heat-conduction equation for the condensed phase and, if the variation of the external conditions (z_j) is known, the problem of nonsteady propellant combustion reduces to the solution of the system

$$\begin{gathered}\frac{\partial T}{\partial t} = \varkappa \frac{\partial^2 T}{\partial x^2} - u(z_j, \varphi_s)\frac{\partial T}{\partial x} \quad (x \leqslant 0)\\ T_s = T_s(z_j, \varphi_s) \ \text{ at } \ x = 0, \ T \to T_0 \ \text{ as } \ x \to -\infty \\ T = T_0 + (T_s^\circ - T_0)\exp\frac{u^\circ \varkappa}{x} \ \text{ at } \ t = 0\end{gathered} \tag{1.9}$$

$$T_F = T_F(z_j, \varphi_s) \tag{1.10}$$

Thus, for known relations (1.1) it is possible to obtain a formal solution of the problem of nonsteady propellant combustion without particularizing the mechanisms of the physicochemical reactions in the flame and in the solid-phase reaction zone. In this sense the theory is phenomenological in character.

There are two possible methods of obtaining relations (1.1). In the first place, using the quasi-stationarity property, it is possible to obtain (1.1) from the experimentally determined steady-state relations

$$u^\circ = u^\circ(z_1, \ldots, z_n, T_0), \ T_s^\circ = T_s^\circ(z_1, \ldots, z_n, T_0), \ T_F^\circ = T_F^\circ(z_1, \ldots, z_n, T_0) \tag{1.11}$$

(steady-state values are denoted by a degree sign) by eliminating the initial temperature T_0 with the aid of the Michelson solution $T_0 = T_s^\circ - (\varphi_s^\circ \varkappa)/u^\circ$ (this method was first proposed by Ya. B. Zel'dovich [4]). The other method of obtaining (1.1) is based on the analytic solution of Eqs. (1.2)-(1.8) [5,6]. However, this method requires that (1.2)-(1.5) be written in exact functional form, which is not possible without a knowledge of the steady-state combustion mechanism and hence without certain assumptions regarding the structure of the flame zone and the kinetics of the chemical reactions in the flame, which considerably restricts the generality of the results obtained.

In principle, both approaches to the problem of obtaining (1.1) are equivalent and if, in the first case, the experiment is sufficiently accurate and, in the second, if the theoretical description of steady-state combustion is successful, give identical results [7]. Problem (1.9) is of independent significance, since it makes possible the determination of the burning rate in the dynamic mode (this problem has been solved in a series of papers [8-16]). At the same time, many of the phenomena observed in connection with nonsteady propellant combustion are determined by the response of the flame to a change in external conditions. In this respect, as first shown quantitatively in the present study, relation (1.10) is important and, as far as a whole series of nonsteady processes is concerned, cannot be ignored.

Returning to relations (1.2)-(1.8), we note that in the general case the net reaction energy $q_s + q_g$ is variable and depends both on the pressure, flow velocity, etc., and on the temperature gradient in the condensed phase. Since

$$q_s + q_g = h_0 - h_F = \sum_s a_j \Delta h_j^\circ - \sum_F a_i \Delta h_i^\circ$$

(h_0, h_F are the enthalpies of formation of the fuel and the end products of the chemical reactions in the flame), using (1.8) for the dependence of the chemical enthalpy of the products of the equilibrium flame on z_j and φ_s we have

$$h_F = h_0 + c_s T_s - c_p T_F - \frac{c_s \varkappa_s \varphi_s}{u} \tag{1.12}$$

2. Starting from the above combustion model, we will examine nonsteady propellant burning in the case when one of the external parameters (for example, pressure) varies according to the law

$$z_j = z_j^\circ + z_{j1} (\cos \omega t;\ \sin \omega t) \tag{2.1}$$

The notation (2.1) implies that either of the functions $\sin \omega t$ or $\cos \omega t$ can be selected.

Going over in (1.19), (1.10), and (2.1) to the dimensionless quantities

$$\xi = \frac{u^\circ x}{\varkappa},\quad \tau = \frac{(u^\circ)^2 t}{\varkappa},\quad \vartheta = \frac{T - T_0}{T_s^\circ - T_0}$$
$$v = \frac{u}{u^\circ},\quad \varphi = \frac{\varphi_s}{\varphi_s^\circ},\quad \Omega = \omega \frac{\varkappa}{(u^\circ)^2}$$
$$Z_j = \frac{z_j}{z_j^\circ},\quad \Delta = \frac{z_{j1}}{z_j^\circ}$$

we reduce the problem to the form

$$\frac{\partial \vartheta}{\partial \tau} + v(Z_j, \varphi) \frac{\partial \vartheta}{\partial \xi} = \frac{\partial^2 \vartheta}{\partial \xi^2} \quad (\xi \leqslant 0) \tag{2.2}$$

$$\vartheta(\tau, -\infty) = 0,\ \vartheta(\tau, 0) = \vartheta_s(Z_j, \varphi),\ \vartheta(0, \xi) = \exp \xi$$
$$\vartheta_F = \vartheta_F(Z_j, \varphi) \tag{2.3}$$
$$Z_j = 1 + \Delta(\cos \Omega\tau;\ \sin \Omega\tau) \tag{2.4}$$

Linearizing systems (2.2)-(2.4) for the first and second approximations with respect to the amplitude of the pressure oscillations $\Delta \ll 1.0$, we have

$$\frac{\partial^2 \vartheta^{(1)}}{\partial \xi^2} - \frac{\partial \vartheta^{(1)}}{\partial \xi} - \frac{\partial \vartheta^{(1)}}{\partial \tau} = v^{(1)} e^{\xi}$$
$$\vartheta^{(1)}(\tau, -\infty) = 0,\ \vartheta^{(1)}(\tau, 0) = \vartheta_s^{(1)}(\tau)$$
$$\vartheta^{(1)}(0, \xi) = 0,\ \vartheta_s^{(1)}(0) = \vartheta_F^{(1)}(0) = v^{(1)}(0) = 0$$
$$X_i(\tau) = \left(\frac{\partial X_i}{\partial Z_j}\right)_\varphi (\cos \Omega\tau;\ \sin \Omega\tau) + \left(\frac{\partial X_i}{\partial \varphi}\right)_{Z_j} \varphi^{(1)}(\tau) \tag{2.5}$$

Here, X_i is any of the functions $v^{(1)}(\tau)$, $v_S^{(1)}(\tau)$, $v_F^{(1)}(\tau)$

$$\frac{\partial^2 \vartheta^{(2)}}{\partial \xi^2} - \frac{\partial \vartheta^{(2)}}{\partial \xi} - \frac{\partial \vartheta^{(2)}}{\partial \tau} = v^{(2)} e^{\xi} + \frac{\partial \vartheta^{(1)}}{\partial \xi} v^{(1)}$$
$$\vartheta^{(2)}(\tau, -\infty) = 0,\ \vartheta^{(2)}(\tau, 0) = \vartheta_s^{(2)}(\tau)$$
$$\vartheta^{(2)}(0, \xi) = 0,\ \vartheta_s^{(2)}(0) = v^{(2)}(0) = \vartheta_F^{(2)}(0) = 0$$
$$X_i(\tau) = \left(\frac{\partial X_i}{\partial \varphi}\right)_{Z_j} \varphi^{(2)} + \frac{1}{2}\left(\frac{\partial^2 X_i}{\partial Z_j^2}\right)_\varphi (\cos^2 \Omega\tau;\ \sin^2 \Omega\tau)$$
$$+ \frac{1}{2}\left(\frac{\partial^2 X_i}{\partial \varphi^2}\right)_{Z_j} [\varphi^{(1)}]^2 + \frac{\partial^2 X_i}{\partial Z_j \partial \varphi} \varphi^{(1)} (\cos \Omega\tau;\ \sin \Omega\tau) \tag{2.6}$$

3. The solution of the problem in the linear approximation (2.5) is found in the form

$$\vartheta^{(1)} = \Theta(\xi) e^{i\Omega\tau},\ v^{(1)}(\tau) = V e^{i\Omega\tau},\ \varphi^{(1)}(\tau) = \Phi e^{i\Omega\tau}$$
$$\vartheta_F^{(1)}(\tau) = \Theta_F e^{i\Omega\tau},\ \vartheta_s^{(1)}(\tau) = \Theta_s e^{i\Omega\tau}$$

where Θ, V, and Φ are the complex amplitudes of the temperature, burning rate, and gradient perturbations, respectively. After performing operations similar to those described in [16], we obtain

$$\Theta(\xi) = \left(\Theta_s - \frac{iV}{\Omega}\right) e^{\varkappa\xi} + \frac{i}{\Omega} V e^{\xi},\ \Theta_s = -\frac{\delta}{k} + \frac{r}{k} V \tag{3.1}$$

$$V = \frac{\nu + \delta(\alpha - 1)}{1 - k + (\alpha - 1)(r - ik/\Omega)}, \quad \Phi = \alpha\Theta_s - \frac{i}{\Omega} V(\alpha - 1)$$
$$\varepsilon\Theta_F = \left(s - \nu\frac{q}{k}\right) + \frac{q}{k}V, \quad \varepsilon = \frac{T_s^\circ - T_0}{T_F^\circ}$$

After isolating the imaginary part [the perturbation $Z_j(\tau)$ is assumed to be sinusoidal], for the linear perturbations we obtain

$$\begin{gathered} Z_j(\tau) = \Delta \sin \Omega\tau \\ v^{(1)}(\tau) = \operatorname{Im}\{V \exp i\Omega\tau\} = |V| \sin(\Omega\tau + \Psi) \\ \varphi^{(1)}(\tau) = \operatorname{Im}\{\Phi \exp i\Omega\tau\} = \frac{k + r - 1}{k}|V| \sin(\Omega\tau + \Psi) - \frac{\delta - \nu}{k} \sin \Omega\tau \\ \varepsilon\vartheta_F^{(1)}(\tau) = \operatorname{Im}\{\Theta_F \exp i\Omega\tau\} = \left(s - \nu\frac{q}{k}\right) \sin \Omega\tau + \frac{q}{k}|V| \sin(\Omega\tau + \Psi) \end{gathered} \tag{3.2}$$

Here,

$$\begin{gathered} |V| = \left(\frac{a^2 + b^2}{c^2 + d^2}\right)^{1/2}, \quad \Psi = \operatorname{arc\,tg} \frac{bc - ad}{ac + bd}, \quad a = \nu + \frac{\delta}{2}\left(\frac{\Omega}{\beta} - 1\right) \\ b = \delta\beta, \quad c = 1 + \left(\frac{\Omega}{\beta} - 1\right)\left(\frac{r}{2} - \frac{k\beta}{\Omega}\right), \quad d = \beta r - \frac{k}{2\Omega}\left(\frac{\Omega}{\beta} - 1\right) \\ \beta = \frac{1}{2}\left[\frac{1}{2}\left(\sqrt{16\Omega^2 + 1} - 1\right)\right]^{1/2}, \quad \alpha = \frac{1}{2}\left(1 + \sqrt{4i\Omega + 1}\right), \quad \delta = \nu r - \mu k \end{gathered}$$

Moreover,

$$\begin{gathered} \nu = \left(\frac{\partial \ln u^\circ}{\partial \ln Z_j}\right)_{T_0}, \quad k = (T_s^\circ - T_0)\left(\frac{\partial \ln u^\circ}{\partial T_0}\right)_{Z_j}, \quad \mu = \frac{1}{T_s^\circ - T_0}\left(\frac{\partial T_s^\circ}{\partial \ln Z_j}\right)_{T_0} \\ r = \left(\frac{\partial T_s^\circ}{\partial T_0}\right)_{Z_j}, \quad s = \left(\frac{\partial \ln T_F^\circ}{\partial \ln Z_j}\right)_{T_0}, \quad q = (T_s^\circ - T_0)\left(\frac{\partial \ln T_F^\circ}{\partial T_0}\right)_{Z_j} \end{gathered} \tag{3.3}$$

The parameters ν, k, μ, r, s, q determine the properties of the fuel and the flame and can be found from the known steady-state relations (1.11). An investigation of combustion in the presence of pressure oscillations ($Z_j = \pi$) has shown [16] that at values of $k > 1.0$ the amplitude $|V|$ of the burning rate is characterized by resonance at the natural frequencies of the solid-phase induction zone $\Omega^* = \sqrt{k}/r$ [at $k < 1.0$ the $V(\Omega)$ dependence is not characterized by resonance]. Obviously, all the conclusions reached in connection with the pressure variation problem also hold for the problem of nonsteady combustion associated with the variation of any other of the Z_n external parameters, provided that under steady-state conditions the Michelson relation $\varphi^\circ = u^\circ(T_s^\circ - T_0)/\varkappa$ is preserved (if the radiation from the gas to the condensed phase is taken into account, the steady-state solution of the heat-conduction equation has another form).

It is clear from (2.2) that the flame-temperature oscillation amplitude dependence must also be resonant in character at the frequency Ω^*.

The investigation of the linear problem of nonsteady propellant combustion in the presence of harmonic pressure oscillations makes it possible to consider the important practical question of the amplification (attenuation) of acoustic pressure waves by the burning surface. The response of the burning surface to a pressure disturbance is usually estimated in terms of a complex quantity – the acoustic admittance of the burning surface [17-19]

$$Y = -\left(\frac{\delta u_g/u_g^\circ}{\delta p/p^\circ}\right) \tag{3.4}$$

where δu_g is the perturbation of the gas velocity normal to the surface of the fuel, and δp is the pressure perturbation. In this case the acoustic wave is amplified by the burning surface if

$$\operatorname{Re} Y < 0 \tag{3.5}$$

Using the continuity equation $\rho_g u_g = \rho_s u_s$ and the equation of state of an ideal gas, we can write expression (3.4) in the form

$$Y = \frac{\delta\pi - \delta v - \varepsilon\delta\vartheta_F}{\delta\pi} \left(\varepsilon = \frac{T_s^\circ - T_0}{T_F^\circ}\right) \tag{3.6}$$

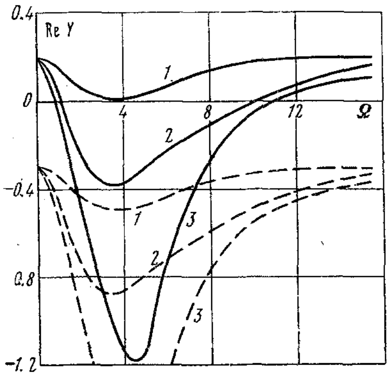

Fig. 1

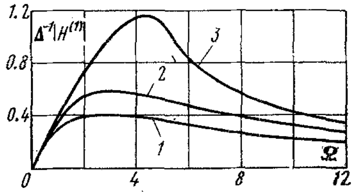

Fig. 2

Hence for the complex amplitudes, using (3.1), we have

$$Y = 1 - s + \nu \frac{q}{k} - \left(1 + \frac{q}{k}\right) V(\Omega) \tag{3.7}$$

Since $V = (a + ib)/(c + id)$, isolating the real part from (3.7), we obtain

$$\operatorname{Re} Y = 1 - s + \nu \frac{q}{k} - \left(1 + \frac{q}{k}\right) \frac{ac + bd}{c^2 + d^2} \tag{3.8}$$

It is clear from (3.8) that the flame characteristics (s, q) have an important influence on the ability of the burning surface to amplify acoustic oscillations, the effect of the parameter $s = (d \ln T_F^\circ / d \ln p)_{T_0}$ being independent of the frequency and at $s > 0$ always tending to produce amplification. This is because in the linear approximation owing to the pressure variation the heat release in the flame is in phase with the pressure oscillations [see Eq. (3.2)] and the Rayleigh criterion for the thermal amplification of acoustic waves is satisfied. (The effect of q on Re Y is a complex one and depends both on the frequency and on the values of the other parameters.)

The dependence of the real part of the acoustic admittance on the dimensionless frequency is presented in Fig. 1 for values of the parameters $\nu = 0.66$, $r = 0.3$, $\mu = 0.1$, $q = 0.1$ (continuous curves 1, 2, 3 correspond to the cases $k = 0.5$, 1.0, 1.5 for $s = 0$, the dashed curves 1, 2, 3 relate to the same values of k, but for $s = 0.5$). Clearly, as the k and s increase, the region of amplification of acoustic waves by the burning surface expands, while the maximum value of $|\operatorname{Re} Y|$ increases, and at $s \sim 0.5$ the propellant amplifies the pressure oscillations over the entire low-frequency range. As was to be expected, the maximum of Re Y corresponds to the dimensionless resonant frequency Ω^*.

We will now consider the question of the possible types of perturbations generated by the flame under nonsteady conditions. Under the influence of varying pressure the flame temperature T_F, the chemical enthalpy of the combustion products h, the gas density ρ, and the flame velocity u_g vary in accordance with quasi-stationary relations (1.1), (1.6), and (1.12). These perturbations of the parameters in the flame front lead to the appearance of a system of waves propagating through the combustion products. In fact, for the adiabatic motion of a chemically frozen, perfect, ideally nonheat-conducting gas we have

$$\begin{gathered} \frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \mathbf{u}) = 0, \quad \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}, \operatorname{grad})\,\mathbf{u} = -\frac{\operatorname{grad} p}{\rho}, \quad p = \rho R T \\ c_p \rho \left(\frac{\partial T}{\partial t} + \mathbf{u} \operatorname{grad} T\right) = \frac{\partial p}{\partial t} + \mathbf{u} \operatorname{grad} p, \quad \frac{\partial (\rho h)}{\partial t} + \operatorname{div}(\rho \mathbf{u} h) = 0 \end{gathered} \tag{3.9}$$

Hence in the one-dimensional case we obtain the following system for small perturbations of the relative values of the pressure $p' = \delta p / p^\circ$, temperature $T' = \delta T / T^\circ$, gas velocity $u' = \delta u / u_g^\circ$, and chemical enthalpy $h' = \delta h / h^\circ$

$$\begin{gathered} \frac{\partial p'}{\partial t} + u_g^\circ \frac{\partial p'}{\partial x} + u_g^\circ \frac{\partial u'}{\partial x} - \frac{\partial T'}{\partial t} - u_g^\circ \frac{\partial T'}{\partial x} = 0 \\ \frac{\partial u'}{\partial t} + u_g^\circ \frac{\partial u'}{\partial x} + \frac{c^2}{\gamma u_g^\circ} \frac{\partial p'}{dx} = 0, \quad \gamma \left(\frac{\partial T'}{\partial t} + u_g^\circ \frac{\partial T'}{\partial x}\right) = (\gamma - 1)\left(\frac{\partial p'}{\partial t} + u_g^\circ \frac{\partial p'}{\partial x}\right) \\ \frac{\partial h'}{\partial t} + u_g^\circ \frac{\partial h'}{\partial x} = 0 \quad (\gamma = c_p / c_v) \end{gathered} \tag{3.10}$$

We find the solution of (3.10) in wave form

$$X_j' = X_j^{(1)} \exp i(\omega t + kx)$$

Then

$$\begin{gathered} p^{(1)}(\omega + k u_g^\circ) + u^{(1)} k u_g^\circ - T^{(1)}(\omega + k u_g^\circ) = 0 \\ p^{(1)} \frac{c^2 k}{\gamma u_g^\circ} + u^{(1)}(\omega + k u_g^\circ) = 0, \quad h^{(1)}(\omega + k u_g^\circ) = 0 \\ p^{(1)}(1 - \gamma)(\omega + k u_g^\circ) + T^{(1)} \gamma (\omega + k u_g^\circ) = 0 \end{gathered} \tag{3.11}$$

The condition of solvability of (3.11) reduces to the vanishing of the determinant

$$\begin{vmatrix} \omega + ku_g^\circ, & ku_g^\circ, & -(\omega + ku_g^\circ), & 0 \\ \dfrac{c^2k}{\gamma u_g^\circ}, & \omega + ku_g^\circ, & 0, & 0 \\ (1-\gamma)(\omega + ku_g^\circ,), & 0, & \gamma(\omega + ku_g^\circ), & 0 \\ 0, & 0, & 0, & \omega + ku_g^\circ \end{vmatrix} = 0$$

Hence we find the wave vectors

$$k_1 = \omega/(c - u_g^\circ), \quad k_2 = -\omega/(c + u_g^\circ), \quad k_3 = -\omega/u_g^\circ$$

Thus, pressure, temperature, chemical energy and velocity waves may exist in the combustion products

$$X_j' = A_j \sin \omega_i \left(k - \frac{x}{c + u_g^\circ}\right) + B_j \sin \omega_i \left(k + \frac{x}{c - u_g^\circ}\right) + C_j \sin \omega i \left(t - \frac{x}{u_g^\circ}\right)$$

$$X_4' = h' = h^{(1)} \exp i\omega \left(t - \frac{x}{u_g^\circ}\right)$$

$$(X_1' = p', \; X_2' = u', \; X_3' = T')$$

In the particular case $\gamma = 1.0$ (so-called "zeroth approximation") the fields of small perturbations of the flow elements decompose into noninteracting components. Expressions (3.11) with $\gamma = 1.0$ yield the wave equations

$$\frac{1}{c^2}\frac{\partial^2 X_{1,2}}{\partial t^2} + 2\frac{\beta}{c}\frac{\partial^2 X_{1,2}}{\partial x \partial t} - (1 - \beta^2)\frac{\partial^2 X_{1,2}}{\partial x^2} = 0 \left(\beta = \frac{u_g^\circ}{c}\right)$$

$$\frac{\partial^2 X_{3,4}}{\partial t^2} = (u^\circ)_g^2 \frac{\partial^2 X_{3,4}}{\partial x^2}$$

which show that in the zeroth approximation the pressure, density, and gas velocity waves propagate with velocity ($c \pm u_g^\circ$) and the temperature and chemical energy waves with the flow velocity u_g°. Since at the same frequency of the acoustic and temperature waves the length of the latter is many times less (under ordinary combustion conditions the wavelength ratio $\lambda_3/\lambda_1 = \beta$ is of the order of 10^{-2}–10^{-3}), near the burning surface the pressure may be assumed to be simply a harmonic function of time $p' = \Delta \sin \omega t$. Then the solutions of the wave equations for T and h take the form

$$X'_{3,4} = |X_F^{(1)}|_{3,4} \sin(\omega t + k_3 x + \Psi_{3,4})$$

where $|X_F^{(1)}|_{3,4}$ are the amplitudes of the perturbations at the flame, and $\Psi_{3,4}$ are the phase shifts. Since the propagation velocities of the T and h waves coincide, this wave system may be treated as the combined wave of the total enthalpy of the products $H' = |H_F^{(1)}| \sin(\omega t + k_3 x + \Psi_H)$. In fact, as the pressure varies, the total enthalpy of the products changes by an amount $\delta H = c_p \delta T_F + \delta h_F$.

Using (1.12), in dimensionless form we obtain

$$\delta H_* = \delta\vartheta_s - \delta\varphi - \delta v \quad (\delta H_* = \delta H/[c_s(T_s^\circ - T_0)]) \tag{3.12}$$

It is interesting to note that the total enthalpy change does not depend on the flame characteristics (s, q) and is wholly determined by the inertia of the solid-phase induction zone. For a sinusoidal pressure variation from (3.12), using (3.2), we obtain

$$\frac{\delta H_*}{\Delta} = \frac{|V|}{k}\sin(\Omega\tau + \Psi) - \frac{V}{k}\sin\Omega\tau$$

This expression may conveniently be represented in the form

$$\delta H_* = |H_F^{(1)}| \sin(\Omega\tau + f)$$

$$|H_F^{(1)}| = \frac{\Delta}{k}\sqrt{|V|^2 - 2\nu|V|\cos\Psi + \nu^2}, \; f = \operatorname{arc\,tg}\frac{|V|\sin\Psi}{|V|\cos\Psi - \nu} \tag{3.13}$$

In Fig. 2 we have plotted the theoretical dependence of the amplitude of the total enthalpy wave (3.13) for the same values of the parameters ν, μ, r as in Fig. 1 (the curves 1, 2, 3 correspond to k = 0.5, 1.0, 1.5). It should be noted that experiments confirm the existence of short temperature waves above the burning surface in the oscillatory mode [20].

4. We will investigate the behavior of the mean flame temperature in the presence of harmonic oscillations of some external parameter controlling combustion. For this purpose it is necessary to consider the solution of the nonsteady burning problem in the second approximation.

In accordance with the theory of nonlinear oscillations in the quadratic approximation the combination frequencies 0 and 2Ω may appear. Since the mean of a harmonic function is equal to zero, only the constant component of the solution of problem (2.6) is of interest. Isolating in (2.6) the terms that do not depend on time, for the constant components we obtain the system

$$\begin{gathered}
\frac{d^2\vartheta_c^{(2)}}{d\xi^2} - \frac{d\vartheta_c^{(2)}}{d\xi} = v_c^{(2)} e^{\xi} + \left[\frac{\partial\vartheta^{(1)}}{\partial\xi} v^{(1)}\right]_c \\
\vartheta_c^{(2)}(\xi \to -\infty) = 0, \quad \vartheta_c^{(2)}(\xi = 0) = \vartheta_{s,c}^{(2)} \\
X_{ic}^{(2)} = \left(\frac{\partial X_i}{\partial\varphi}\right)_{Z_j} \varphi_c^{(2)} + \frac{1}{2}\left(\frac{\partial^2 X_i}{\partial Z_j^2}\right)_{\varphi} [\sin^2\Omega\tau]_c \\
+ \frac{1}{2}\left(\frac{\partial^2 X_i}{\partial\varphi^2}\right)_{Z_j} [(\varphi^{(1)})^2]_c + \frac{\partial^2 X_j}{\partial Z_j \partial\varphi} [\varphi^{(1)} \sin\Omega\tau]_c
\end{gathered} \tag{4.1}$$

Here, the subscript c denotes the constant component of the functions.

We write expressions for the constant components in (4.1) in terms of the complex amplitudes of the linear perturbations of the burning rate V and surface temperature Θ_s [see (3.1)]. Since for a sinusoidal perturbation $Z_j(\tau)$

$$\begin{gathered}
\vartheta^{(1)}(\xi, \tau) = \operatorname{Im}\{\Theta(\xi) \exp i\Omega\tau\} = \frac{1}{2i}\{\Theta(\xi) e^{i\Omega\tau} - \Theta^*(\xi) e^{-i\Omega\tau}\} \\
v^{(1)}(\tau) = \operatorname{Im}\{V \exp i\Omega\tau\} = \frac{1}{2i}\{V e^{i\Omega\tau} - V^* e^{-i\Omega\tau}\}
\end{gathered}$$

and so on, it is easy to calculate the quantities

$$\begin{gathered}
\left[\frac{\partial\vartheta^{(1)}}{\partial\xi} v^{(1)}\right]_c = \frac{1}{4}\left\{\alpha^*\Theta_s^* V e^{\alpha^*\xi} + \alpha\Theta_s V^* e^{\alpha\xi} + \frac{i|V|^2}{\Omega}(\alpha^* e^{\alpha^*\xi} - \alpha e^{\alpha\xi})\right\} \\
[(\varphi^{(1)})^2]_c = \frac{1}{2}\left\{|\alpha|^2\left[|\Theta_s|^2 + \frac{i}{\Omega}(\Theta_s V^* - \Theta_s^* V) + \frac{|V|^2}{\Omega}\right]\right. \\
\left. + \frac{i}{\Omega}(\alpha^*\Theta_s^* V - V^*\Theta_s\alpha) - \frac{|V|^2}{\Omega^2}(\alpha^* + \alpha) + \frac{|V|^2}{\Omega^2}\right\} \\
[\varphi^{(1)} \sin\Omega\tau]_c = \frac{1}{4}\left\{(\alpha\Theta_s + \alpha^*\Theta_s^*) + \frac{i}{\Omega}[V^*(\alpha^* - 1) - V(\alpha - 1)]\right\}
\end{gathered} \tag{4.2}$$

Here, an asterisk denotes the complex conjugate. Integrating the heat-conduction equation from (4.1) with respect to ξ from 0 to $-\infty$, we find the relation (4.3).

$$\varphi_c^{(2)} = \vartheta_c^{(2)} + v_c^{(2)} + {}^1\!/_4(\Theta_s^* V + \Theta_s V^*) \tag{4.3}$$

Using (4.3) and (4.1), we can, in principle, express the value of the constant component of the flame temperature $\vartheta_{F,c}^{(2)}$ in the second approximation in terms of the amplitudes V and Θ_s known from the solution of the linear problem. However, the general expression obtained is extremely clumsy and not amenable to analysis. Accordingly, we shall confine ourselves to a consideration of the function $\vartheta_{F,c}^{(2)}$ near resonance. (We recall that for a propellant burning under the influence of harmonic perturbations of some function Z_j resonance effects are possible only at k > 1.0).

It is known that in the resonance region the amplitude of the first harmonic is proportional to the cube root of the amplitude of the driving force, while the amplitude of the second harmonic and the constant component are proportional to the square of the amplitude of the first harmonic. Accordingly, we can estimate the order of magnitude of the constant terms in the expansions of the functions $v_c^{(2)}$, $\vartheta_{s,c}^{(2)}$, $\vartheta_{F,c}^{(2)}$ from (4.1). Retaining in these expansions terms of the order of $\Delta^{2/3}$ (Δ is the small amplitude of the oscillations of the quantity Z_j), near resonance we have

$$X_{i,c}^{(2)} \approx \left(\frac{\partial X_i}{\partial \varphi}\right)_{Z_j} \varphi_c^{(2)} + \frac{1}{2}\left(\frac{\partial^2 X_i}{\partial \varphi^2}\right)_{Z_j} [(\varphi^{(1)})^2]_c \tag{4.4}$$

In the linear approximation after making similar estimates we obtain

$$V \approx \left(\frac{\partial v}{\partial \varphi}\right)_{Z_j} \Phi, \quad \Theta_s \approx \left(\frac{\partial \vartheta_s}{\partial \varphi}\right)_{Z_j} \Phi$$

Hence, using (3.3), we obtain

$$\Theta_s = \frac{r}{k} V \tag{4.5}$$

Using (4.3), (4.4), and (4.5), after transformations we find

$$v_c^{(2)} = -r\frac{|V|^2}{2} - \frac{1}{2}[(\varphi^{(1)})^2]_c\left[k\frac{\partial^2 \vartheta_s}{\partial \varphi^2} + (k-1)\frac{\partial^2 v}{\partial \varphi^2}\right] \tag{4.6}$$

$$\vartheta_{F,c}^{(2)} = -\frac{[(\varphi^{(1)})^2]_c}{2}\left[(k+r-1)\frac{\partial \vartheta_F}{\partial \varphi}\left(\frac{\partial^2 v}{\partial \varphi^2} + \frac{\partial^2 \vartheta_s}{\partial \varphi^2}\right) - \frac{\partial^2 \vartheta_F}{\partial \varphi^2}\right]$$
$$-\frac{|V|^2}{2}\frac{r(k+r-1)}{k}\frac{\partial \vartheta_F}{\partial \varphi} \tag{4.7}$$

where

$$[(\varphi^{(1)})^2]_c = \frac{|V|^2}{2}\left\{|\alpha|^2\left(\frac{r^2}{k^2} + \frac{1}{\Omega^2}\right) + \frac{i}{\Omega}\frac{r}{k}(\alpha^* - \alpha) - \frac{1}{\Omega^2}(\alpha^* + \alpha - 1)\right. \tag{4.8}$$

Equations (4.6) and (4.7) can be converted to the form

$$v_c^{(2)} = \frac{1}{2}[(\varphi^{(1)})^2]_c\left(\frac{\partial k}{\partial \varphi}\right)_{Z_j}\frac{1}{k+r-1} - r\frac{|V|^2}{2}$$
$$\varepsilon\vartheta_c^{(2)} = \frac{1}{2}[(\varphi^{(1)})^2]_c\left(\frac{\partial q}{\partial \varphi}\right)_{Z_j}\frac{1}{k+r-1} - r\frac{q}{k}\frac{|V|^2}{2}$$

Since

$$\left(\frac{\partial k}{\partial \varphi}\right)_{Z_j} = \frac{\partial k/\partial T_0}{\partial \varphi/\partial T_0} = \frac{(r-1)k - k^2 + (T_s{}^\circ - T_0)^2(\partial^2 u^\circ/\partial T_0{}^2)}{k+r-1}$$
$$\left(\frac{\partial q}{\partial \varphi}\right)_{Z_j} = \frac{(r-1)q - q^2 + (T_s{}^\circ - T_0)^2(\partial^2 T_F{}^\circ/\partial T_0{}^2)/T_F{}^\circ}{k+r-1}$$

and $q > 0$, $r < 1.0$, we have $(\partial k/\partial \varphi) < 0$, $(\partial q/\partial \varphi) < 0$ and, consequently, near resonance $v_c^{(2)}$ and $\varepsilon\vartheta_{F,c}^{(2)}$ are negative, i.e., in an acoustic field with frequency close to the natural frequency of the solid-phase induction zone the mean flame temperature and the burning rate are reduced as compared with their steady-state values, the reduction being proportional to the square of the perturbation amplitude. The change in mean flame temperature can be attributed to the fact that some of the energy is removed from the flame with the T and h waves and irreversibly lost.

LITERATURE CITED

1. Ya. B. Zel'dovich, "Theory of combustion of propellants and explosives," Zh. Éksperim. i Teor. Fiz., 12, No. 11-12 (1942).
2. B. V. Novozhilov, "Nonsteady combustion of propellants with variable surface temperature," Zh. Prikl. Mekhan. i Tekh. Fiz., No. 1 (1967).
3. Yu. A. Gostintsev, P. F. Pokhil, and L. A. Sukhanov, "Complete system of equations for the nonstationary processes associated with the combustion of a propellant in a semiclosed space," Dokl. Akad. Nauk SSSR, 195, No. 1 (1970).
4. Ya. B. Zel'dovich, "Propellant burning rate at variable pressure," Zh. Prikl. Mekhan. i Tekh. Fiz., No. 3 (1964).
5. S. S. Novikov and Yu. S. Ryazantsev, "Theory of propellant burning stability," Zh. Prikl. Mekhan. i Tekh. Fiz., No. 1 (1965).
6. A. G. Istratov and V. B. Librovich, "Propellant burning stability," Zh. Prikl. Mekhan. i Tekh. Fiz., No. 5 (1964).

7. M. Summerfield, L. H. Caveny, Battista, N. Kubota, Yu. Gostintsev, and H. Isoda, "Theory of dynamic extinguishment of solid propellants with special reference to nonsteady heat feedback law," J. Space Rockets, 8, No. 3 (1971).
8. B. V. Novozhilov, "Transient processes in solid propellant combustion," Zh. Prikl. Mekhan. i Tekh. Fiz., No. 5 (1962).
9. V. B. Librovich, "Ignition of solid propellants and explosives," Zh. Prikl. Mekhan. i Tekh. Fiz., No. 6 (1963).
10. Yu. A. Gostintsev and A. D. Margolin, "Nonsteady burning of thin propellant slabs," Zh. Prikl. Mekhan. i Tekh. Fiz., No. 5 (1964).
11. Yu. A. Gostintsev and A. D. Margolin, "Nonstationary combustion of a powder under the action of a pressure pulse," Fiz. Goreniya i Vzryva, No. 2 (1965).
12. S. S. Novikov and Yu. S. Ryazantsev, "Extinction of propellant near the contact with a metal," AIAA J., 8, No. 2 (1970).
13. B. V. Novozhilov, "Nonlinear oscillations of propellant burning rate," Zh. Prikl. Mekhan. i Tekh. Fiz., No. 5 (1966).
14. A. G. Istratov, V. B. Librovich, and B. V. Novozhilov, "An approximate method in the theory of nonsteady propellant burning rates," Zh. Prikl. Mekhan. i Tekh. Fiz., No. 3 (1964).
15. Yu. A. Gostintsev, "Method of reduction to ordinary differential equations in problems of the nonstationary burning of solid propellants," Fiz. Goreniya i Vzryva, 3, No. 3 (1967).
16. B. V. Novozhilov, "Propellant combustion in the presence of harmonically varying pressure," Zh. Prikl. Mekhan. i Tekh. Fiz., No. 6 (1965).
17. A. D. Margolin, I. B. Svetlichnyi, P. F. Pokhil, and A. S. Tsirul'nikov, "Variation of the acoustic admittance of the burning surface of a propellant," Zh. Prikl. Mekhan. i Tekh. Fiz., No. 1 (1970).
18. F. T. McClure, R. W. Hart, and J. F. Bird, "Solid propellant rocket motors as acoustic oscillators," in: Solid Propellant Rocket Research, Academic Press, New York (1960).
19. S. S. Novikov and Yu. S. Ryazantsev, "Interaction of acoustic waves with the burning surface of condensed systems," Zh. Prikl. Mekhan. i Tekh. Fiz., No. 2 (1966).
20. H. Krier, M. Summerfield, H. B. Mathes, and E. W. Price, "Entropy waves produced in oscillatory combustion of solid propellants," AIAA J., 7, No. 11 (1969).